

Worksheet for 2021-08-30

Conceptual questions

Question 1. Find a parametrization for the curve $y^2 = x^3$ that traces out the entire curve (not just part of it!).

Question 2. Let $x = f(t)$, $y = g(t)$ be a parametric curve such that $g'(3) = 0$. What can you conclude (if anything) about the tangent line at $t = 3$?

Question 3. True or false: for a parametric curve $x = f(t)$, $y = g(t)$, we have $d^2y/dx^2 = \frac{d^2y/dt^2}{d^2x/dt^2}$.

Computations

Problem 1. Find a Cartesian equation for the parametric curve $x = t^3 + t$, $y = t^2 + 2$. Hint: compute x^2 .

Find the slope of this curve at the point $(10, 6)$. If you remember implicit differentiation, try using that on the Cartesian equation and check that you get the same answer.

Problem 2 (Stewart §10.2.54). Compute the arclength of the “astroid” $x = \cos^3 t$, $y = \sin^3 t$ depicted in Figure 1. (Stewart §10.2.34 asks you for the area.)

Problem 3. There are two points on the curve

$$x = 2t^2, y = t - t^2, -\infty < t < \infty$$

where the tangent line passes through the point $(10, -2)$. Find these two points.

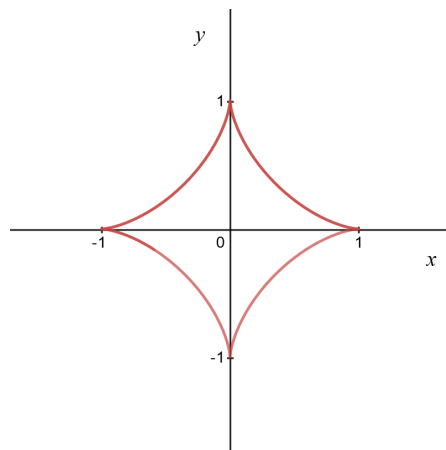


FIGURE 1. Problem 3